THE SOLUTION OF The five Problems,

Which was proposed to *James Paterson*, in the *Mathematicall Dispute*, by *George Liddell* Student of *Philosophy* and *Mathematicks* in the *Marischall Colledge* of *Aberdeen*, *December* 4. 1684.

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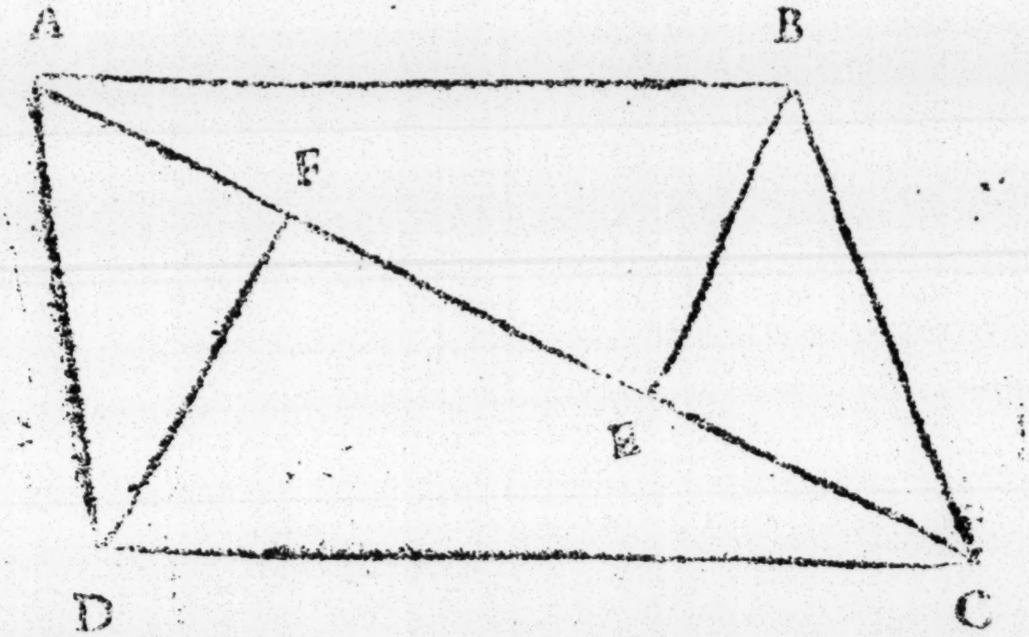
The Solution of the Five Problems.

Problem 1. There is a Plot or Parcel of Ground A B C D surveyed and ploted, containing 23 *Acres*. 1 *R* 24 *Pole*. I demand by what *Scale* of *Pearches* this *Area* was found. I desire that *Demonstrative & Geometrice* you perform the same, with all the subsequent *Problems*

Ans. *The same parcel of Ground was ploted by a scale of* 10 *Chain*, *or* 40 *Pole in an Inch.*



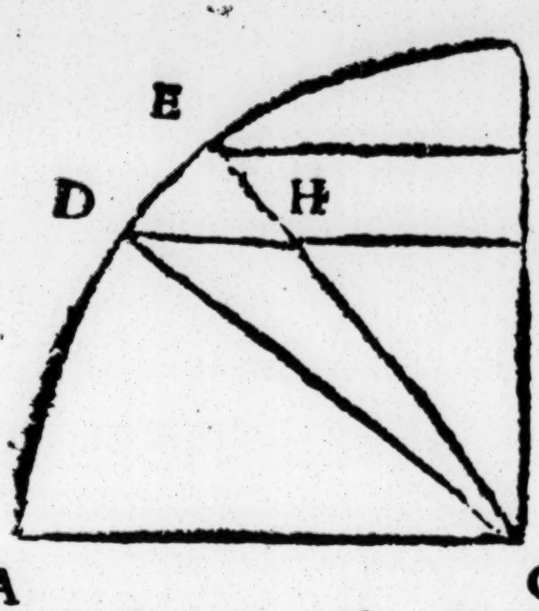
Const. *Let there be an parcell of Ground,* as A B C D, *having drawn the Diagonall-Line* A C *which is found to be* 26 *Chain, and the perpendiculars* B E *and* D F 18 *Chain the half whereof being* 9, *multiplyed in* 26,

giveth 2304 *square Chains*, (*by the* 24 *of the first of* Euclid.) *Where* 160 *square Chains makes one Acre, and therefore the Area of this Trapezium doth contain* 23 *Acres* 1 *rood*, 24 *Pole but more particularly the scale may be found by having two like figures, with a scale propper to one of them We may find a scale propper to the other for all like figures; are in a duplicat reason of their homologall sides*, (*by* 8 : 19 : 20 : *prop.* 6 Euclid.)

Prob 2. There is a *Semi-Circle* whose *Diameter* being 100, and in the same there is described a *Sector*, whose *Base* upon the *Peripherie* doth contain 30 *deg.* I demand the *Area* of a *Segment* of that *Semi-Circle*, with two straight *Lines* parallell to the *Diameter* equal to the *Sector*. *Const & Dem.*

Let there be a quadrant of a Circle A B C, *in whose Peripherie set off* A D 37 *deg* 30 *min and* E B, *equal to the same then shall* E D *be* 15 *deg. then from* D *to the Center* C, *draw a line also. from* E *to* C, *also, from* D *to* F, *parallell to* A C. *and from* E *to* G, *parallell to the same; I say that the mixt Triangle,* D C E, *is equall to the mixt Trapezium* D E G F.

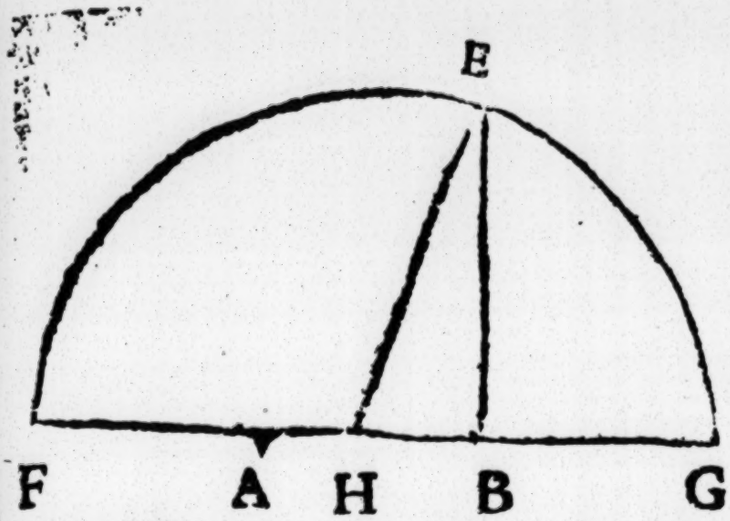
Demonst.



Demonst. *The Tri-angle* D C F, *and* E C G, *are equal, for* E G, *and* F C, *are equal, being each of them the fines of* 37 *deg* 30 *min. Again,* C G, *and* D F, *are equal, being the fines of* 52 *deg.* 30 *min. with the two right Angles at* G *and* F; *therefore, the two Triangles* D F C, *and* E G C, *are equal, (by the* 4 Prop. *of the firſt of* Euclid.) *But* H F C, *being common, taken away, there remaineth* D H C, *and* E H, F G, *equal, (by* 3 Ax. 1 Euclid.) *unto which if ye add* D H E, *then ſhal the mixed Tri-angle* D C E *be equal to the mixed Quadrilateral* D E G F, *(by* 2 Ax. 1 Euclid.) *ſeeing therefore, the mixed Triangle, being the half of the Sector, whoſe Baſe is* 30 *deg. and the Segment of the Semi-Circle, with two ſtraight lines to the Diameter, the double of* D F G E: *therefore, as halfs are to halfs, ſo wholes to wholes, (by* 6 Ax. 1 Euclid.) *which was to be demonſtrated.*

Prob. 3. There is a right *Angled plain Triangle*, the difference of the *Sides* being 25, a middle proportionall between the *Sides* being 37. I demand the quantity of the *Sides*, *Conſt. & Demonſt.*

Conſt.



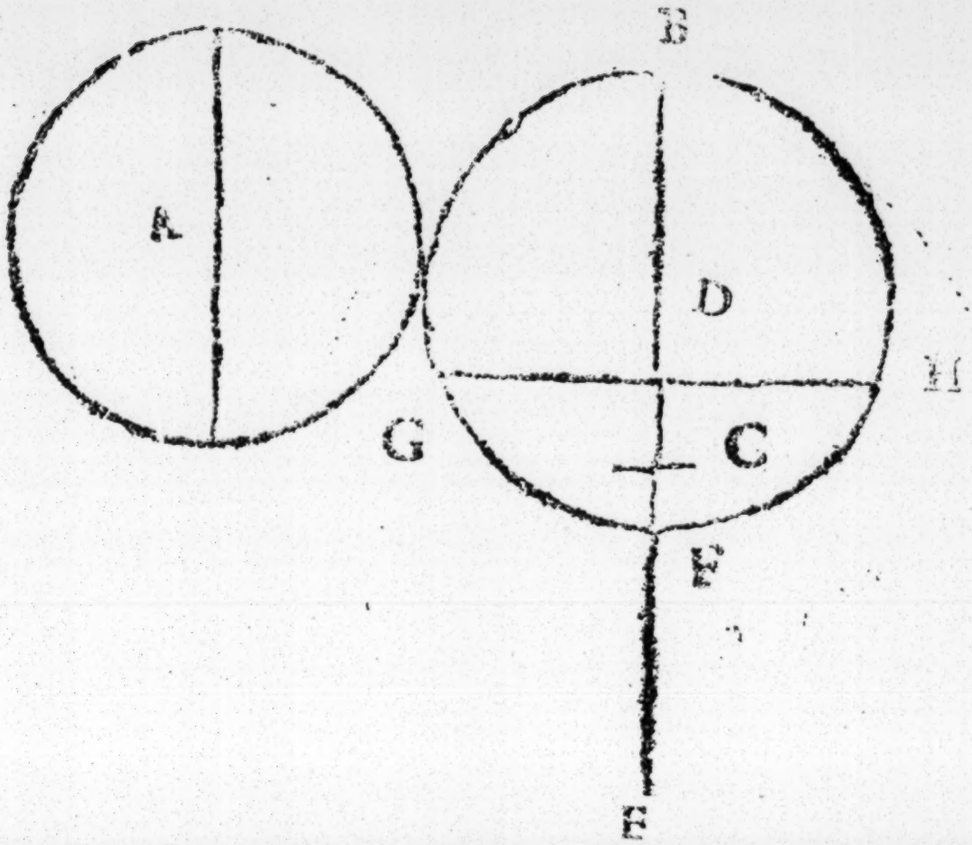
Conſt.

Let A B *be* 25, *the difference of the ſides, and the half thereof* 12, 5. *then from* D, *erect a perpendicular as* B E, 37, *equal to the middle proportional betwixt the ſides, then from the Center* H *deſcribe a Semi-Circle, meeting with* A B, *being produced in* F *and* G. *I ſay* F B, *and* B G, *ſhall be the ſides of the Rectang required.*

Demonſt. *For ſeeing* A H, *equal to* H B, *and* F H *to* H G, *ſubſtracting* A H *and* H B, *the remainder is* F A *and* B G *ſhall be equal wherefore the exceſs of* F B, *above* B G *is* A B *given, but* B E *is a middle proportionall betwixt* F B *and* B G. *The ſides of the Rectang. required are,* F B *and* B G.

Prob. 4. There is a *Sphere* or *Glob* as A, whoſe *Diameter* is 100, there is another *Glob* B, divyded into two *Segments*, parallell to the *Diameter*, the greater *Segment* being equal to the *Glob* A, the *Altitude* of the ſame *Segment* being 75. I demand the *Diameter* of thar *Glob*, *Conſt. & Demonſt:*

Let



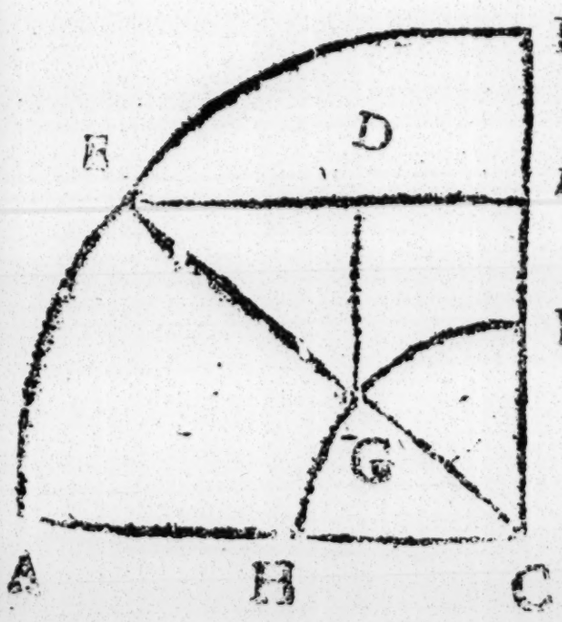
Let there be given the Sphere A. *to whose Diameter be equall* B C, *and in the same let there be taken a point as* D. *let* B D *be* 75, *for the Altitude of the greater Segment, equall to the Sphere* A. *Having produced the line* B C, *say, as the square of* B D 5625. *is to the square of* B C 10000. *so* B C, 100. *to* D E 177, 77 *Divide the same in three equall parts, of which* D F *let be one of the parts, which is* 59, 26: *which added to* 75. *gives* 132, 26 *for the Diameter* B F, *about the which, having described a Sphere as being cut by a Plain right angled to the Diameter through the point* D, *it shall give the portion* G B H. *in the Altitude of* B D 75, *which shall be equal to the given Sphere* A.

Demonst. *For it is demonstrated as* B E : B D : : *segment* G B H : *Sphere whose Diameter is* B D, *because it is made as the square of* B D. *to*

the

the ſquare of B C, *ſo the line* B C, *to* B E. *therefore the ſolus made of the ſquare of* D, *in the line* B E, *ſhall be equall to the Cub of* B C. *Wherefore, the Cub of* B C, *to the Cub of* B D *is as* B E *to* B D; *but ſpheres are as the Cubs of their Diameters therefore the Sphere of the Diameter* B C *is to the ſphere of the Diameter* B D, *as the line* B E *to the line* B D, *that is, as the ſegment* G B H. *to the ſphere whoſe Diameter is* B D, *therefore the ſphere of the Diameter* B C, *that is the ſphere* A *given, is equal to the ſegment* G B H, *quod faciendum erat.*

Prob. 5. There is a *Quadrant* in which the greater *Semidiameter* is given. and a Point in the ſame *Quadrant* I would know the leſſer *Semidiameter* of an *Ellypſis*, which ſhall paſs through the Point given. Again both the *Semidiameters* being given, the greater and the leſſer, I would know by what Point in the parallell of 40 the *Ellypſis* ſhall paſs.



Conſt. *Let there be a Quadrant drawn* A B C, *and a Point in the ſame given as* D; *through which an Ellypſis is to be deſcribed for finding the leſſer Diameter through the Point* D, *draw a parallell to* A C *as* E F, *then*

from E *to the Center, draw a line as* E C, *then from* D *draw a parallell to* B C, *cutting* E C *in* G, *then* C G *being equall to* C H, *ſhall be the leſſer Semidiameter of the Ellypſis.*

Demonſt, *As* E D : D F : : E G : G C, *by the* (10 Prop. 6 Euclid.) *&* componendo, (*by* def. 10. 5. Euclid.) *as* E F : D F : : E C : G C, *in numeris, as the ſine of* E F 50 *deg.* ſine 766, *to* D E, *ſine of* 10 *deg.* ſine 173. *ſo Rad.* 1000 *to* G C *or* H C 22584. *Again having both the Semidiameters, we may find the point in the parallell of* 40 *deg, by which the Ellypſis ſhall paſs, as the point* D. *He that deſireth more let him conſult* Apollonius Perigæus, *de* 5 ſect. Coni. *This Problem is of ſingular uſe in the Orthographical Projection, of the ſphere in Plano.*

And now to conclude, I having expected the ſolution of theſe five *Problems*, from the fourth of *December* 1684; to the middle of *February* in this preſent year 1685: and having found no ſolution of them, neither by Pen nor Print: Therefore, I having thought expedient to give a perfect ſolution of them my ſelf, to the publick aſpect and view of all ingenious and learned PHILOMATHISTS.

FINIS.